Fuzzy Network Control Systems based upon a Real-time Implementation

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Abstract:

Fuzzy control, as a global stable strategy based upon piece wise integration, allows a switched approximation for nonlinear behavior, where variations are expected during time delays due to inherent network communication. Time delays variations may be determined from communication system or by a scheduling strategy, through their stochastic behavior. Variations may be studied considering bounded known behavior, or through classic delay estimation; in either case results would be only switchable for known cases.
This condition is tackled using conditions of processes period selection, making possible the switching under a precise known delay stage, and its related estimation. A Fuzzy Takagi-Sugeno approach has been proposed, considering the time delays behavior. Using this approach, several experiments have been developed, ranging from simulations based upon MATLAB to implementations using real hardware in the loop. Through several years, many case studies have been studied and tested, such as a three tanks benchmark, a three conveyor belts example, an aerodynamic airplane simulation, a magnetic levitation system, and an helicopter modeling. In this chapter, the basics of such an approach are presented, explained along with two selected case studies from the list above: a three tanks benchmark and the helicopter modeling.
The main goal of this chapter is to show how a Network Control System (NCS) can be implemented as a feasible strategy for time delays variations. Moreover, it is reviewed a strategy based upon a dynamic linear time invariant model, managing the relationship between frequencies of agents in a distributed system, and using LQR control scheme to bring the system into a linear region. Hence, the objective of this chapter is to present a reconfiguration control strategy for NCSs, making use of a Fuzzy Takagi-Sugeno Model Predictive Control. The dynamic behavior of such a NCS is modeled using a real-time implementation of the scheduling algorithm.

1. Introduction

The design and implementation of Real-Time Distributed Systems (RTDS) has two major trends of choice. On one hand, a first major trend makes emphasis on the tasks and the processors that execute them. For this option, and in the case of single processors, there are very well proven scheduling algorithms such as Rate Monotonic (RM), or Earliest Deadline First (EDF) (Liu, 2000). Other authors, such as Regehr and Stankovic (2001), Feng et al. (2002), and Mok et al. (2004), have considered multiple real-time resources or components, and a hierarchical composition of schedulers has proven to be a prolific research line. Other approaches, such as Easwaran et al. (2006-1,2) and Chakraborty et al. (2006), have proposed frameworks for real-time scheduler composition, which have been constantly developed and produced sounded schedulability analysis for hierarchical real-rime systems. Furthermore, the schedulability analysis for multiple processor systems has produced global schedulability analysis, as described by Baruah et al. (2008), Guan et al. (2008), or Anand et al. (2008).

Nevertheless, and on the other hand, the second major trend has proposed to implement a RTDS based upon a Real-time network, using a Computer Area Network (CAN), and a communication protocol such as Time Triggered Protocol (TTP) or Flexible Time Triggered (FTT) as proposed by Almeida et al. (2004). The global scheduling for Real-time distributed systems is still an open research area (Menendez et al., 2010); while there are important contributions to global multiprocessor scheduling, most of the approaches to this analysis has not taken into account the communications network, and hence, do not entirely apply to distributed systems. On the counterpart, when the emphasis is made on the Real-time network, the processor scheduling is practically left aside. While in both cases the system might have a blend of both approaches, and actually work quite well, most of the times this suggestion involves to have a global strategy, that includes both real-time elements as basic components of a RTDS. This allows to compose a complete real-time distributed system, with real-time scheduling algorithms in every processor, and a real-time network. This proposed approach will eventually allow the design of a global RTDS scheduler, and the possibility to have a global point for validating that all time constraints of a RTDS are to be fulfilled. In order to demonstrate the feasibility of such a strategy, several case studies, consisting of a NCS, have been presented before.

Distributed systems are widely used in industry and research. These systems fulfill critical mission and long-running applications. Some characteristics of the computation performed by distributed systems are either capacity to maintain consistency, or recovering without suspending their execution. Distributed systems should complete time restrictions, coherence, adaptability, and stability, among others. In order that distributed systems achieve their overall objectives, it is necessary for all their components to properly exchange information through communication media. Therefore, the communication mechanism plays an important role on the stability and performance of distributed systems, seen as control systems implemented over a communication network (or simply, NCS), as proposed by Lian et al. (2006). Further, network scheduling has been a priority in the design of a NCS, when a group of components or agents are linked together through the available network resources. If there is no coordination among such agents, data transmissions may occur simultaneously, and it has to back off to avoid collisions or bandwidth violations. This results in transmissions with delay or even failure to comply their real-time deadlines. The necessity of an adequate scheduling control algorithm, trying to minimize this loss of system performance, has been proposed by Branicky et al. (2003). Nevertheless, in this approach there is no global scheduler that guarantees an optimal system performance, as it has been shown by Menendez et al. (2009B).

Mainly, since the communication network introduces a number of issues that are not properly dealt, Lian et al. (2001, 2002) designed methodologies for networked nodes (agents of the system) to generate proper control actions, and optimally utilize communication bandwidth. Thus, the effectiveness of the control system depends on the sampling rate. Hence, it is very important to considerate either sampling periods or frequency transmission to obtain better system performance (Esquivel-Rosas et al., 2010). On the other hand, a consensus algorithm may be proposed. The basic idea of using a consensus algorithm is to impose similar dynamics on the information states of each agent involved in a dynamical system. In networks of agents, consensus means to reach an agreement regarding a certain quantity of interest that depends on the state of all agents, as described by Ren et al. (2007), Olfati-Saber et al. (2007), and Hayashi et al. (2008).

In this chapter, control reconfiguration is presented as an available approach for fault coverage, in order to keep system performance. Reconfiguration is pursued as response of time delay modification rather than fault appearance, although this is the basis for control reconfiguration using the Takagi-Sugeno approach.

Several other approaches have been proponed for managing time delays within control laws by different research groups. For instance, Nilsson (1998) proposes the use of a time delay scheme integrated to a reconfigurable control strategy, based upon a stochastic methodology. Another approach by Jiang et al., (1999) use time delays as uncertainties, which modify pole placement of a robust control law. Izadi et al. (1999) present an interesting case of fault tolerant control approach, related to time delays coupling. Blanke et al. (2003) have proposed reconfigurable control from the point of view of structural modification, establishing a logical relation between dynamic variables and the respective faults. Finally, Benítez-Pérez et al. (2005) and Thompson (2004) have considered that reconfigurable control performs a combined modification of system structure and dynamic response, with the advantage of bounded modifications over system response.

The approach here considers time delays due to communication as deterministic measured variables, as well as actuator fault presence by modification of system parameters. This affects local control with two conditions: loosing local peripheral elements and the related time delays. The control law views time delays as a result of deterministic reconfigurable communications, based upon Fuzzy Takagi-Sugeno approach, as described by Mendez et al. (2010).

The control design and stability analysis of NCSs have been studied in recent years. The main advantages of this kind of systems are their low cost, small volume of wiring, distributed processing, simple installation, and maintenance and reliability, as presented by Menendez et al. (2009).

In a NCS, a key issue is the effect of network-induced delays in system's performance. Delays can be constant, time-varying, or even random. This depends on the scheduler, network type, hardware architecture, operating systems, and so on. For example, Nilsson (1998) has analyzed several important features of NCSs. He introduced models for delays in NCS, first as a fixed delay, later as an independently random variable, and finally as a Markov process. He also introduces optimal stochastic control theorems for NCSs, based upon the independently random and Markovian delay models. Walsh et al. (1999) introduced static and dynamic scheduling policies for transmission of sensor data in a continuous-time LTI system. They introduced the notion of the *maximum allowable transfer interval* (MATI), which is the longest time after which a sensor should transmit data. These authors derived bounds of the MATI, such that the NCS is stable. This MATI ensures that the Lyapunov function of the system under consideration is strictly decreasing at all times. Zhang (2001) extended the work of Walsh et al. (1999), developing a theorem which ensures the decrease of a Lyapunov function for a discrete-time LTI system at each sampling instant, by using two different bounds. These results are less conservative, because they do not require the system's Lyapunov function to be strictly decreasing at all time.

Tzes (2003) introduced a number of different linear matrix inequality (LMI) tools for analyzing and designing optimal switched NCSs. Zhu et al. (2008) took into consideration both the network-induced delay and the time delay in the plant, proposing a controller design method by using the delay-dependent approach. An appropriate Lyapunov functional candidate is used to obtain a memoryless feedback controller. This is derived by solving a set of Linear Matrix Inequalities (LMIs). Wang et al. (2007) modeled the network induced delays of NCSs as interval variables governed by a Markov chain. Using the upper and lower bounds of the delays, a discrete-time Markovian jump system with norm-bounded uncertainties is presented to model the NCSs. Based on this model, the $H\infty$ state feedback controller can be constructed via a set of LMIs. Recently, Fridman et al., (2003) introduced a new (descriptor) model transformation of delay-dependent stability for systems with time-varying delays in terms of LMIs. They also refined recent results on delay-dependent $H\infty$ control, and extended them to the case of time-varying delays. Based upon this review, this paper defined a model that integrated the time delays for a class of nonlinear system, and therefore, this paper presented a supervisory control for NCSs using fuzzy control and considering time delay induced by the computer network. The stability analysis of this paper is based on LMI. Moreover, the emergence of a smart sensor and an actuator technology removes the need for centralized control with feedback loops to dumb peripheral actuators, replacing it with a databus connection. This gives an autonomous actuator installation (Masten, 1997) as well as local control, self-calibration, health monitoring, and reconfiguration availabilities.

Further, several other approaches for managing time delay within control laws have been studied for different research groups. Nilsson (1998) proposed the use of a time delay scheme integrated to a reconfigurable control strategy, based upon a stochastic methodology. Wu (1997) proposed a reconfiguration strategy based upon a performance measure from a parameter estimation fault diagnosis procedure. Another approach has been proposed by Jiang et al. (1999), in which time delays are used as uncertainties, modifying pole placement of a robust control law. Izadi-Zamanabadi et al. (1999) presented an interesting view of a fault tolerant control approach, related to time delay coupling. Blanke et al., (2003) have studied reconfigurable control from the point of view of structural modification. Benitez-Perez et al., (2005) proposed a combined modification of system structure and dynamical systems.

In the approach of this chapter, time delays due to communication are taken as deterministic measured variables. Also, the actuator fault presence is considered by modifying the B matrix, in order to propose a Fuzzy Takagi-Sugeno control with two conditions: loose of local peripheral elements and the related time delays. Here, like in the approach by Mendez-Monroy et al. (2009), time delays result of deterministic reconfigurable communications based upon a scheduling algorithm. These time delays are a structural consequence determined by the insertion of new elements within communication channels, due to fault appearance. In fact, fault presence is taken into account as the lost of the related peripheral element, specifically, sensor or actuator elements.

2. Fuzzy Network Control Systems

Fuzzy control is developed in terms of bounded time delays, allowing the universal approximation to feasible for this kind of problems (Quiñones-Reyes et al., 2010). Thus, the plant model is considered to have the following dynamics:

$$\begin{aligned} x(k+1) &= a^p x(k) + B^p u(k) \\ y &= c^p x(k) \end{aligned} \tag{1}$$

where:

- $a^p \in \Re^{nxn}$,
- $c^p \in \Re^{nx1}$,
- $B^p \in \Re^{nx1}$ are matrices related to the plant, and
- $x(k)$, $u(k)$ and $y(k)$ are states, inputs, and outputs, respectively.

In particular, B^P is defined as:

$$B^p = \sum_{i=1}^{N} \rho_i B_i \sum_{j=1}^{M} \int e^{-a^p(t-\tau)} \mathrm{d}\tau \tag{2}$$

where:

- $\rho_i = \{0,1\}$, $\sum_{i=1}^{N} \rho_i = 1$
- *N* is the total number of possible faults,
- *M* is the involved time delays from each fault,
- τ_{j-1}^i and τ_j^i are current communication time delay, $\sum_{j=1}^{M} \tau_j^i \le T$ where ***T*** is the total transport

delay of the cycle and depends of the faults scenarios.

Thus, B_i is an array:

$$B_i = \begin{bmatrix} b_1 \\ b_2 \\ 0_i \\ \vdots \end{bmatrix}$$

where:

- $b_1 \rightarrow b_N$ are the elements conformed at the input of the plant (such as actuators), and
- 0_i is the lost element due to local actuator fault.

B^p represents only one scenario (see Equation 6). A further definition of a current B_i^p considers local actuator faults and related time delays:

$$B_i^p = B_i \sum_{j=1}^{M} \int_{\tau_j^i}^{\tau_{j-1}^i} e^{-a^p(t-\tau)} d\tau \qquad (3)$$

For simplicity, B_i^p is used in order to describe local linear plants.

From this representation, a fuzzy plant is defined taking into consideration each time delay, fault case, and the related fuzzy rules:

$$r_i : \text{if } x_1 \text{ is } \mu_{1i} \text{ and } x_2 \text{ is } \mu_{2i} \text{ and…and } x_l \text{ is } \mu_{li} \text{ then } a_i^p x(k) + B_i^p u(k) \qquad (4)$$

where:

- $\{x_1 x_2 .... x_l\}$ are current state measures,
- l is the number of states,
- $i = \{1,..., N\}$ is one of the fuzzy rules,
- N is the number of the rules which is equal to the number of possible faults, and
- μ_{ij} are the related membership functions, which are Gaussian defined as:

$$\mu_{ij}(y_\alpha, u_\alpha) = \exp\left(-\left(\frac{y_\alpha - c_{ij}^{y}}{\sigma_{ij}^{y}} \right)^2 - \left(\frac{u_\alpha - c_{ij}^{u}}{\sigma_{ij}^{u}} \right)^2 \right) \qquad (5)$$

where c_{ij} and σ_{ij} are parameters to be tuned.

Such a representation of the plant, as an integrated system with the control, is thus based on centre of area de-fuzzification method. From this representation of a global nonlinear system, it is necessary to define a global stability condition as a result of this fuzzy system. This is given considering fuzzy logic control approach. The result allows the integration of nonlinear stages and transitions to basically a group of linear plants. As from the point of view of the approach, taking the input of the plant as consequent, this is defined as Fuzzy MPC, as follows:

$$u_i = (S_i^T \overline{Q} S_i + \overline{R})^{-1} S_i \overline{Q} (w - p_i) \quad (6)$$

where:

- w are the future set points,
- u_i is the control output,
- Q and R are positive definite weight matrices defined as:

$$\overline{Q} = Diag(Q)$$
$$\overline{R} = Diag(R)$$

- S represents the effect of future outputs and from the integration to antecedent representation of the fuzzy logic system (see Equation 8), over N_p and N_c horizons defined by the user:

$$S = \begin{bmatrix} S_{Np_1} & S_{Np_1 - 1} & \cdots & 0 \\ S_{Np_1+1} & S_{Np_1} & S_{Np_1-1} & \\ \vdots & \vdots & \ddots & \\ S_{Np_2} & S_{Np_2-1} & \cdots & S_{Np_2-N_c} \end{bmatrix}, \quad (7)$$

where:

$$s_j = y_0 \qquad \forall j \le n_d,$$
$$s_j = \sum_{i=1}^{Na} a_i S_{j-i} + \sum_{i=1}^{Nb} B^{p_i} \qquad j > n_d \quad (8)$$
$$p_i = \sum_{j=1}^{Na} a_j p_{i-j} + \sum_{j=1}^{Nb} b_j B_j^p u(k - j - n_d + i) + c$$

Figure 1 shows how the horizons take place in time.

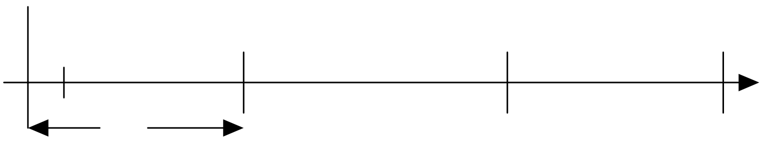

Figure 1. Time horizons with respect to time delays. Horizon samples and *k* sampling time.

In Figure 1, N_a and N_b are the horizon samples, k is the sampling time, l is the related time delay within the sampling time, n_d is the minimum discrete dead-time. Notice that in Equation 8, the parameters of the plant are presented as a_j and B_i^p where:

$$b_j = f(B_j^p)$$

and:

$$B_i^p = \int_{t_i}^{t_{i+1}} e^{(\Delta\Delta - \tau)} d\tau B$$

From the integration of the antecedent part of the representation of Fuzzy system (see Equation 4) it is obtained that:

$$D_i(y_\alpha,u_{\alpha'})=\prod_{j=1}^{N_p}\mu_{ij}(y_\alpha,u_{\alpha'})=\prod_{j=1}^{N_p}\exp\left(-\left(\frac{y_\alpha-c_{ij}^{y}}{\sigma_{ij}^{y}}\right)^2-\left(\frac{u_{\alpha'}-c_{ij}^{u}}{\sigma_{ij}^{u}}\right)^2\right) \tag{9}$$

$$\alpha=\begin{cases}0 & j>N_p\\ j & 1\le j\le N_p\end{cases} \qquad \alpha'=\begin{cases}0 & 1\le j\le N_p\\ j & j>N_p\end{cases}$$

where:

- N_p is the number of possible inputs for the fuzzy plant,
- y is the output of the plant, and
- u is the plant input.

For the antecedent part of fuzzy control Ω_i :

$$\Omega_i(y_\alpha,u_{\alpha'})=\prod_{j=1}^{N_A}\mu_{ij}(y_\alpha,u_{\alpha'})=\prod_{j=1}^{N_A}\exp\left(-\left(\frac{y_\alpha-c_{ij}^{y}}{\sigma_{ij}^{y}}\right)^2-\left(\frac{u_{\alpha'}-c_{ij}^{u}}{\sigma_{ij}^{u}}\right)^2\right) \tag{10}$$

$$\alpha\begin{cases}0 & j>N_A\\ j & 1\le j\le N_A\end{cases} \qquad \alpha'\begin{cases}0 & 1\le j\le N_A\\ j & j>N_A\end{cases}$$

where N_A is the number of possible inputs for the fuzzy controller. In this case, fault conditions are presented as the results of local time delays, more than the actual loss of current measure. Thus, the plant representation is given as follows, considering time delays:

$$x(k+1)=\frac{\sum_{i=1}^{N}D_i(y_\alpha,u_{\alpha'})\left(a_i x(k)+B_i^p u(k)\right)}{\sum_{i=1}^{N}D_i(y_\alpha,u_{\alpha'})} \tag{11}$$

where N is the number of rules, and the plant input is defined as considering time delays expressed in Equation 10:

$$u(k)=\frac{\sum_{i=1}^{N}\Omega_i(y_\alpha,u_{\alpha'})\left(S_k^T\overline{Q}S_k+\overline{R}\right)^{-1}S_k\overline{Q}(w-p_k)}{\sum_{i=1}^{N}\Omega_i(y_\alpha,u_{\alpha'})} \tag{12}$$

Substituting in Equation 11:

$$x(k+1)=\frac{\sum_{i=1}^{N}D_i(y_\alpha,u_{\alpha'})\left(a_i x(k)+B_i^p\frac{\sum_{i=1}^{N}\Omega_i(y_\alpha,u_{\alpha'})\left(S_k^T\overline{Q}S_k+\overline{R}\right)^{-1}S_k\overline{Q}(w-p_k)}{\sum_{i=1}^{N}\Omega_i(y_\alpha,u_{\alpha'})}\right)}{\sum_{i=1}^{N}D_i(y_\alpha,u_{\alpha'})} \tag{13}$$

On the other hand, in order to establish valid horizons, and considering time delays and failures, a MPC strategy is used. Thus, the cost function in MPC is defined as:

$$J=\sum_{i=1}^{N_p} B_i^p\left(ref_i-y_i\right)^2+\sum_{i=1}^{N_A}\delta_i\left(u_i\right)^2 \tag{14}$$

where ref_i and y_i are the reference and output values, respectively. Equation 14 can be rewritten as:

$$J=\sum_{k=1}^{N_p} B_k^p\left(ref_k-Cx(k-1)\right)^2+\sum_{k=1}^{N_A}\delta_k\left(u_k\right)^2 \tag{15}$$

Considering the variables x and u_i, defined in Equations 11 and 12:

$$J=\sum_{k=1}^{N_p} B_k^p\left(ref_k-C\frac{\sum_{i=1}^{N} D_i(y_\alpha,u_{\alpha'})\left(a_i x(k-2)+B_i^p u(k-2)\right)}{\sum_{i=1}^{N} D_i(y_\alpha,u_{\alpha'})}\right)^2+\sum_{k=1}^{N_A}\delta_k\left(\frac{\sum_{i=1}^{N}\Omega_i(y_\alpha,u_{\alpha'})\left(S_k^T\overline{Q}S_k+\overline{R}\right)^{-1}S_k\overline{Q}(w-p_k)}{\sum_{i=1}^{N}\Omega_i(y_\alpha,u_{\alpha'})}\right)^2 \tag{16}$$

Since the values of Q, S, and R are defined as positive definite matrices in Equation 6, it is now necessary to obtain the partial derivatives for each variable, in order to get the optimal values. From Equation 9, the partial derivatives of D_i are obtained with respect to c_{ij} and σ_{ij}:

$$\frac{\partial D_i}{\partial c_{\mathrm{ij}}^{\mathrm{y}}}=-2\sum_{k=1}^{N_p}\left(\frac{y_\alpha-c_{\mathrm{ik}}^{\mathrm{y}}}{\left(\sigma_{\mathrm{ik}}^{\mathrm{y}}\right)^2}\right)\exp\left(-\left(\frac{y_\alpha-c_{\mathrm{ik}}^{\mathrm{y}}}{\sigma_{\mathrm{ik}}^{\mathrm{y}}}\right)^2\right)\prod_{j=1,\,\mathrm{j}\neq\mathrm{k}}^{N_p}\exp\left(-\left(\frac{y_\alpha-c_{\mathrm{ij}}^{\mathrm{y}}}{\sigma_{\mathrm{ij}}^{\mathrm{y}}}\right)^2-\left(\frac{u_{\alpha'}-c_{\mathrm{ij}}^{\mathrm{u}}}{\sigma_{\mathrm{ij}}^{\mathrm{u}}}\right)^2\right) \tag{17}$$

$$\frac{\partial D_i}{\partial c_{\mathrm{ij}}^{\mathrm{u}}}=-2\sum_{k=1}^{N_p}\left(\frac{u_{\alpha'}-c_{\mathrm{ik}}^{\mathrm{u}}}{\left(\sigma_{\mathrm{ik}}^{\mathrm{u}}\right)^2}\right)\exp\left(-\left(\frac{u_{\alpha'}-c_{\mathrm{ik}}^{\mathrm{u}}}{\sigma_{\mathrm{ik}}^{\mathrm{u}}}\right)^2\right)\prod_{j=1,\,\mathrm{j}\neq\mathrm{k}}^{N_p}\exp\left(-\left(\frac{y_\alpha-c_{\mathrm{ij}}^{\mathrm{y}}}{\sigma_{\mathrm{ij}}^{\mathrm{y}}}\right)^2-\left(\frac{u_{\alpha'}-c_{\mathrm{ij}}^{\mathrm{u}}}{\sigma_{\mathrm{ij}}^{\mathrm{u}}}\right)^2\right) \tag{18}$$

Analogously, for $\sigma_{\mathrm{ij}}^{\mathrm{y}}$ and $\sigma_{\mathrm{ij}}^{\mathrm{u}}$:

$$\frac{\partial D_i}{\partial \sigma_{\mathrm{ij}}^{\mathrm{y}}}=-\frac{1}{2}\sum_{k=1}^{N_p}\frac{\left(y_\alpha-c_{\mathrm{ik}}^{\mathrm{y}}\right)^2}{\left(\sigma_{\mathrm{ik}}^{\mathrm{y}}\right)^3}\exp\left(-\left(\frac{y_\alpha-c_{\mathrm{ik}}^{\mathrm{y}}}{\sigma_{\mathrm{ik}}^{\mathrm{y}}}\right)^2\right)\prod_{j=1,\,\mathrm{j}\neq\mathrm{k}}^{N_p}\exp\left(-\left(\frac{y_\alpha-c_{\mathrm{ij}}^{\mathrm{y}}}{\sigma_{\mathrm{ij}}^{\mathrm{y}}}\right)^2-\left(\frac{u_{\alpha'}-c_{\mathrm{ij}}^{\mathrm{u}}}{\sigma_{\mathrm{ij}}^{\mathrm{u}}}\right)^2\right) \tag{19}$$

$$\frac{\partial D_i}{\partial \sigma_{\mathrm{ij}}^{\mathrm{u}}}=-\frac{1}{2}\sum_{k=1}^{N_p}\frac{\left(u_{\alpha'}-c_{\mathrm{ik}}^{\mathrm{u}}\right)^2}{\left(\sigma_{\mathrm{ik}}^{\mathrm{u}}\right)^3}\exp\left(-\left(\frac{u_{\alpha'}-c_{\mathrm{ik}}^{\mathrm{u}}}{\sigma_{\mathrm{ik}}^{\mathrm{u}}}\right)^2\right)\prod_{j=1,\,\mathrm{j}\neq\mathrm{k}}^{N_p}\exp\left(-\left(\frac{y_\alpha-c_{\mathrm{ij}}^{\mathrm{y}}}{\sigma_{\mathrm{ij}}^{\mathrm{y}}}\right)^2-\left(\frac{u_{\alpha'}-c_{\mathrm{ij}}^{\mathrm{u}}}{\sigma_{\mathrm{ij}}^{\mathrm{u}}}\right)^2\right) \tag{20}$$

Applying these, results:

$$\frac{\partial J}{\partial c_{\mathrm{ij}}^{\mathrm{y}}}=\frac{\partial J}{\partial D_i}\frac{\partial D_i}{\partial c_{\mathrm{ij}}^{\mathrm{y}}}=2C\sum_{k=1}^{N_p} B_k^p\left(ref_k-Cx(k-1)\right)\left(\frac{\sum_{i=1}^{N}\left(a_i x(k-2)+B_i^p u(k-2)\right)-Nx(k-1)}{\sum_{i=1}^{N} D_i(y_\alpha,u_{\alpha'})}\right)$$
$$\times\left(-2\sum_{k=1}^{N_p}\left(\frac{y_\alpha-c_{\mathrm{ik}}^{\mathrm{y}}}{\left(\sigma_{\mathrm{ik}}^{\mathrm{y}}\right)^2}\right)\exp\left(-\left(\frac{y_\alpha-c_{\mathrm{ik}}^{\mathrm{y}}}{\sigma_{\mathrm{ik}}^{\mathrm{y}}}\right)^2\right)\prod_{j=1,\,\mathrm{j}\neq\mathrm{k}}^{N_p}\exp\left(-\left(\frac{y_\alpha-c_{\mathrm{ij}}^{\mathrm{y}}}{\sigma_{\mathrm{ij}}^{\mathrm{y}}}\right)^2-\left(\frac{u_{\alpha'}-c_{\mathrm{ij}}^{\mathrm{u}}}{\sigma_{\mathrm{ij}}^{\mathrm{u}}}\right)^2\right)\right) \tag{21}$$

and applying the same for $c_{\mathrm{ij}}^{\mathrm{u}}$:

$$\frac{\partial J}{\partial c_{\mathrm{ij}}^{\mathrm{u}}}=\frac{\partial J}{\partial D_i}\frac{\partial D_i}{\partial c_{\mathrm{ij}}^{\mathrm{u}}}=2C\sum_{k=1}^{N_p}B_k^p\left(ref_k-Cx(k-1)\right)\left(\frac{\sum_{i=1}^{N}\left(a_i x(k-2)+B_i^p u(k-2)\right)-Nx(k-1)}{\sum_{i=1}^{N}D_i(y_\alpha,u_{\alpha'})}\right)$$
$$\times\left(-2\sum_{k=1}^{N_p}\left(\frac{u_{\alpha'}-c_{\mathrm{ik}}^{\mathrm{u}}}{\left(\sigma_{\mathrm{ik}}^{\mathrm{u}}\right)^2}\right)\exp\left(-\left(\frac{u_{\alpha'}-c_{\mathrm{ik}}^{\mathrm{u}}}{\sigma_{\mathrm{ik}}^{\mathrm{u}}}\right)^2\right)\prod_{j=1,\, j\neq k}^{N_p}\exp\left(-\left(\frac{y_\alpha-c_{\mathrm{ij}}^{\mathrm{y}}}{\sigma_{\mathrm{ij}}^{\mathrm{y}}}\right)^2-\left(\frac{u_{\alpha'}-c_{\mathrm{ij}}^{\mathrm{u}}}{\sigma_{\mathrm{ij}}^{\mathrm{u}}}\right)^2\right)\right)\quad(22)$$

Analogously for $\sigma_{\mathrm{ij}}^{\mathrm{y}}$ and $\sigma_{\mathrm{ij}}^{\mathrm{u}}$:

$$\frac{\partial J}{\partial \sigma_{\mathrm{ij}}^{\mathrm{y}}}=\frac{\partial J}{\partial D_i}\frac{\partial D_i}{\partial \sigma_{\mathrm{ij}}^{\mathrm{y}}}=2C\sum_{k=1}^{N_p}B_k^p\left(ref_k-Cx(k-1)\right)\left(\frac{\sum_{i=1}^{N}\left(a_i x(k-2)+B_i^p u(k-2)\right)-Nx(k-1)}{\sum_{i=1}^{N}D_i(y_\alpha,u_{\alpha'})}\right)$$
$$\times\left(-\frac{1}{2}\sum_{k=1}^{N_p}\frac{\left(y_\alpha-c_{\mathrm{ik}}^{\mathrm{y}}\right)^2}{\left(\sigma_{\mathrm{ik}}^{\mathrm{y}}\right)^3}\exp\left(-\left(\frac{y_\alpha-c_{\mathrm{ik}}^{\mathrm{y}}}{\sigma_{\mathrm{ik}}^{\mathrm{y}}}\right)^2\right)\prod_{j=1,\, j\neq k}^{N_p}\exp\left(-\left(\frac{y_\alpha-c_{\mathrm{ij}}^{\mathrm{y}}}{\sigma_{\mathrm{ij}}^{\mathrm{y}}}\right)^2-\left(\frac{u_{\alpha'}-c_{\mathrm{ij}}^{\mathrm{u}}}{\sigma_{\mathrm{ij}}^{\mathrm{u}}}\right)^2\right)\right)\quad(23)$$

$$\frac{\partial J}{\partial \sigma_{\mathrm{ij}}^{\mathrm{u}}}=\frac{\partial J}{\partial D_i}\frac{\partial D_i}{\partial \sigma_{\mathrm{ij}}^{\mathrm{u}}}=2C\sum_{k=1}^{N_p}B_k^p\left(ref_k-Cx(k-1)\right)\left(\frac{\sum_{i=1}^{N}\left(a_i x(k-2)+B_i^p u(k-2)\right)-Nx(k-1)}{\sum_{i=1}^{N}D_i(y_\alpha,u_{\alpha'})}\right)$$
$$\times\left(-\frac{1}{2}\sum_{k=1}^{N_p}\frac{\left(u_{\alpha'}-c_{\mathrm{ik}}^{\mathrm{u}}\right)^2}{\left(\sigma_{\mathrm{ik}}^{\mathrm{u}}\right)^3}\exp\left(-\left(\frac{u_{\alpha'}-c_{\mathrm{ik}}^{\mathrm{u}}}{\sigma_{\mathrm{ik}}^{\mathrm{u}}}\right)^2\right)\prod_{j=1,\, j\neq k}^{N_p}\exp\left(-\left(\frac{y_\alpha-c_{\mathrm{ij}}^{\mathrm{y}}}{\sigma_{\mathrm{ij}}^{\mathrm{y}}}\right)^2-\left(\frac{u_{\alpha'}-c_{\mathrm{ij}}^{\mathrm{u}}}{\sigma_{\mathrm{ij}}^{\mathrm{u}}}\right)^2\right)\right)\quad(24)$$

A similar procedure is used to obtain the partial derivatives with respect to c_{ij} and σ_{ij}, using Equation 24. The optimization procedure of *Q* and *R* are left to the use of this multivariable optimization procedure, since these are design variables.

3. Frequency transition model

Alternatively, the analysis of scheduling approximation is taken in terms of dynamic frequency transitions, where time bounding is desired for full understanding of real-time distributed systems.

Let a distributed system with *n* nodes (components or agents) that perform a task t_i with period p_i, and consumption c_i, $i=1,2,\ldots$. The distributed system dynamics are modeled as a linear time-invariant system, whose state variables $x_1,x_2,\ldots,x_n$ are frequencies of transmisión $f_i=\frac{1}{p_i}$ of the involved *n* nodes. Here, it is assumed that there is a relationship between frequencies $f_1,f_2,\ldots,f_n$ and external input frequencies $u_1,u_2,\ldots,u_n$, which serve as coefficients of the linear system:

$$\dot{x}=Ax+Bu$$
$$y=Cx\quad(25)$$

where:

- $A \in \Re^{nxn}$ is the matrix of relationships between frequencies of the nodes,
- $B \in \Re^{nxn}$ is the scale frequencies matrix,
- $C \in \Re^{nxn}$ is the matrix with frequencies ordered,
- $x \in \Re^{n}$ is a real frequencies vector, $y \in \Re^{n}$ is the vector of output frequencies.

The input $u = k(r - x) \in \Re^{n}$ is a function of reference frequencies and real frequencies of the nodes in the distributed system. It is important to note that relations between the frequencies of the n nodes is schedulable with respect to the use of processors, that is:

$$U = \sum_{i=1}^{n} \frac{c_i}{p_i}$$

where c_i is consume, and p_i are the period of task ti.

Therefore, it is possible to control the system through the input vector u, such that the outputs y are in a non-linear region L, where the system is schedulable. This means that during the time evolution of the system, the output frequencies could be stabilized by a controller within the schedulability region L. This region could be unique, or a set of subregions L_i, in which each y_i converges, as defined by:

$$\beta_1 \le y_1 \le \alpha_1 \ , \beta_2 \le y_2 \le \alpha_n \ , \cdots, \beta_n \le y_n \le \alpha_n$$

Each α_i and β_i belongs to minimum and maximum frequencies respectively to the node n_i, which vary according to every particular case study. Each node of the system starts with a frequency f_i, and the LQR controller modifies the period $f_i = \frac{1}{p_i}$ of each task into the schedulable region L, which is within α and β.regions. After the control action, the real frequency f_i of the node n_i is modified to $f_i^{'}$. This means that p_i in the time t_0 changes to $p_i^{'}$ at time t_1 to converge in a region where the system performance is close to optimal. Figure 2 shows the time diagram of system dynamics. The LQR controller modifications set the task periods into region L.

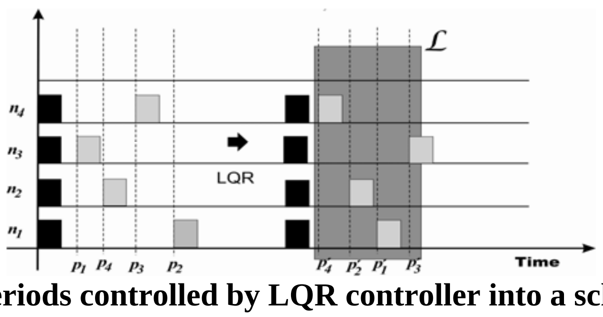

Figure 2. Task periods controlled by LQR controller into a schedulable region.

The objective of controlling the frequency is to achieve coordination through the convergence of values.

3.1 Matrix coefficients proposal

Let $a_{ij} \in A$ given by a function of minimal frequencies f_m of node i, and $b_{ij} \in B$ given by a function of maximal frequencies f_x:

$$a_{ij} = \varphi\left(f_m^1, f_m^2, \ldots, f_m^n\right) = \varphi\left(f_m\right) \qquad\qquad b_{ij} = \psi\left(f_x^1, f_x^2, \ldots, f_x^n\right) = \psi\left(f_x\right)$$

The control input is given as a function of the minimal frequencies, and the real frequencies of node i, that is:

$$u = h(r - x) = k\left(f_m - f_r\right)$$

where f_m, f_r, f_x are the vectors:

$$f_m = \left[f_m^1, f_m^2, \ldots, f_m^n\right]^T \qquad f_r = \left[f_r^1, f_r^2, \ldots, f_r^n\right]^T \qquad f_x = \left[f_x^1, f_x^2, \ldots, f_x^n\right]^T$$

Thus:

$$\dot{x} = Ax + Bu = Af_r + B\left(k\left(f_m - f_r\right)\right)$$

Let us consider a NCS with four sensors. The coefficients of the matrices for such a system (25) are shown as follows:

$$a_{ij} = \begin{cases} \dfrac{\bar{\lambda}\left(f_m^1, f_m^2, f_m^3, f_m^4\right)}{f_m^i} & i = j \\[2ex] \dfrac{f_m^j}{f_m^i} & i \neq j \end{cases} \qquad b_{ij} = \begin{cases} \dfrac{1}{f_x^i} & i = j \\[2ex] 0 & i \neq j \end{cases} \qquad c_{ij} = \begin{cases} 1 & i = j \\[2ex] 0 & i \neq j \end{cases}$$

$\bar{\lambda}\left(f_m^1, f_m^2, f_m^3, f_m^4\right)$ is the greatest common divisor of the minimum frequencies, referred as $\bar{\lambda}$. The system (25) can be re-written as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \dfrac{\bar{\lambda}}{f_m^1} & \dfrac{f_m^2}{f_m^1} & \dfrac{f_m^3}{f_m^1} & \dfrac{f_m^4}{f_m^1} \\[2ex] \dfrac{f_m^1}{f_m^2} & \dfrac{\bar{\lambda}}{f_m^2} & \dfrac{f_m^3}{f_m^2} & \dfrac{f_m^4}{f_m^2} \\[2ex] \dfrac{f_m^1}{f_m^3} & \dfrac{f_m^2}{f_m^3} & \dfrac{\bar{\lambda}}{f_m^3} & \dfrac{f_m^4}{f_m^3} \\[2ex] \dfrac{f_m^1}{f_m^4} & \dfrac{f_m^2}{f_m^4} & \dfrac{f_m^3}{f_m^4} & \dfrac{\bar{\lambda}}{f_m^4} \end{bmatrix} \begin{bmatrix} f_r^1 \\ f_r^1 \\ f_r^1 \\ f_r^1 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{f_x^1} & 0 & 0 & 0 \\[2ex] 0 & \dfrac{f_r^2}{f_x^2} & 0 & 0 \\[2ex] 0 & 0 & \dfrac{1}{f_x^3} & 0 \\[2ex] 0 & 0 & 0 & \dfrac{1}{f_x^4} \end{bmatrix} \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix} \left(\begin{bmatrix} f_m^1 \\ f_m^2 \\ f_m^3 \\ f_m^4 \end{bmatrix} - \begin{bmatrix} f_r^1 \\ f_r^2 \\ f_r^3 \\ f_r^4 \end{bmatrix} \right)$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_r^1 \\ f_r^2 \\ f_r^3 \\ f_r^4 \end{bmatrix} \qquad \textbf{(26)}$$

4. First Case Study

The first case study presented in this chapter is based upon a three tanks problem, as shown in Figure 3 (Benitez-Perez et al., 2010).

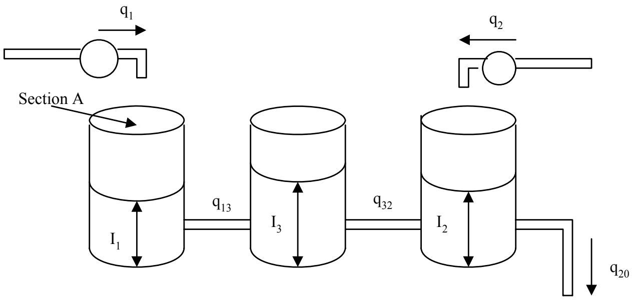

Figure 3. Three tanks representation.

The problem is composed of three tanks that contain a liquid, with identical cross section A. These tanks are coupled by two pipes with cross section of A_2. Two pumps (not shown in Figure 3) supply two inflows q_1 and q_2, powered by DC motors. The liquid level in each tank is measured and reported as I_1, I_2 and I_3. Figure 4 shows the computer network used for this problem. This computer system has a sampling period of 80 ms, and a nominal communication time delay of 20 ms.

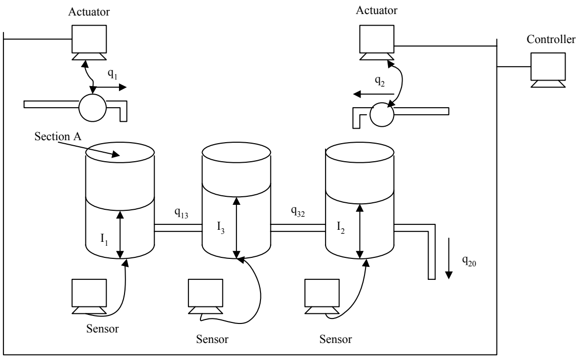

Figure 4. Three tanks representation based upon a Computer Network.

A common representation of this system is represented in Equation 27, as a state space representation. Such a representation takes into account the inherent non-linearities of the model.

$$x(k+1)=\begin{bmatrix} -\frac{q_{13}x}{A} \\ \frac{q_{32}x-q_{20}x}{A} \\ \frac{q_{13}x-q_{32}x}{A} \end{bmatrix}+\begin{bmatrix} \frac{1}{A} & 0 \\ 0 & \frac{1}{A} \\ 0 & 0 \end{bmatrix}U \tag{27}$$

where $x=[l_1 \quad l_2 \quad l_3]^T$, $y=[l_1 \quad l_2]^T$ and $u=[q_1 \quad q_2]^T$ and

$$q_{ij}=\mu_{ij}A*\text{signum}(l_i-l_j)\sqrt{\left(2g|l_i-l_j|\right)} \tag{28}$$

$$q_{20}=\mu_{20}A\sqrt{2gl_2} \tag{29}$$

Here, μ_{ij} represent the outflow coefficients.

In terms of a fuzzy representation, three rules are given regarding the possible number of faults. Here, the possible number of faults is 2, due to just two actuators are available.

$$r_1 \text{ if } x_1 \text{ is } A^c_{11} \text{ and } x_2 \text{ is } A^c_{21} \text{ and } x_3 \text{ is } A^c_{31} \text{ then } a^p{}_1 x(k) + B^p_1 u(k) \tag{30}$$

$$r_2 \text{ if } x_1 \text{ is } A^c_{12} \text{ and } x_2 \text{ is } A^c_{22} \text{ and } x_3 \text{ is } A^c_{32} \text{ then } a^p{}_2 x(k) + B^p_2 u(k) \tag{31}$$

$$r_3 \text{ if } x_1 \text{ is } A^c_{13} \text{ and } x_2 \text{ is } A^c_{23} \text{ and } x_3 \text{ is } A^c_{33} \text{ then } a^p{}_3 x(k) + B^p_3 u(k) \tag{32}$$

In this case B^k_i, it is defined as:

$$B^p_1 = B_1\left(\int_{0.0}^{0.03} e^{-a^p_1(t-\tau)} d\tau + \int_{0.03}^{0.08} e^{-a^p_1(t-\tau)} d\tau\right)$$

$$B^p_2 = B_2\left(\int_{0.0}^{0.04} e^{-a^p_2(t-\tau)} d\tau + \int_{0.04}^{0.08} e^{-a^p_2(t-\tau)} d\tau\right)$$

$$B^p_3 = B_3\left(\int_{0.0}^{0.05} e^{-a^p_3(t-\tau)} d\tau + \int_{0.05}^{0.08} e^{-a^p_3(t-\tau)} d\tau\right)$$

These three plant representations are modified just as B matrix, where reconfiguration is performed. Here, two integrals are presented, due to important time delays of related faults. For instance, B_2^p has a time delay of 0.04, and B_3^p has a time delay of 0.05 seconds.

In the same way, three control laws are used:

$$r_1 \text{ if } x_1 \text{ is } A^c_{11} \text{ and } x_2 \text{ is } A^c_{21} \text{ and } x_3 \text{ is } A^c_{31} \text{ then } u(k) = -g_1 x(k) \tag{33}$$

$$r_2 \text{ if } x_1 \text{ is } A^c_{12} \text{ and } x_2 \text{ is } A^c_{22} \text{ and } x_3 \text{ is } A^c_{32} \text{ then } u(k) = -g_2 x(k) \tag{34}$$

$$r_3 \text{ if } x_1 \text{ is } A^c_{13} \text{ and } x_2 \text{ is } A^c_{23} \text{ and } x_3 \text{ is } A^c_{33} \text{ then } u(k) = -g_3 x(k) \tag{35}$$

Based upon these equations, a final closed loop equation is:

$$x(k+1) = \frac{\sum_{i=1,j=1}^{3} h_i w_j \left(\left(a_i - c_i g_j B^k_j\right) x(k) + B^k_j g_j ref\right)}{\sum_{i=1,j=1}^{3} h_i w_j} \tag{36}$$

Since this representation is obtained from the stability proposal presented in Section 3, an optimization toolbox from MATLAB is used to define current values of the l matrices.

4.1 Implementation Approach

This case study has been implemented over a computer network simulation, using True-Time (Cervin et al., 2003; Benitez-Perez et al., 2010). This simulation consists of a CSMA/CA CAN network, integrated over ten nodes. Figure 5 shows a typical time diagram of this case study.

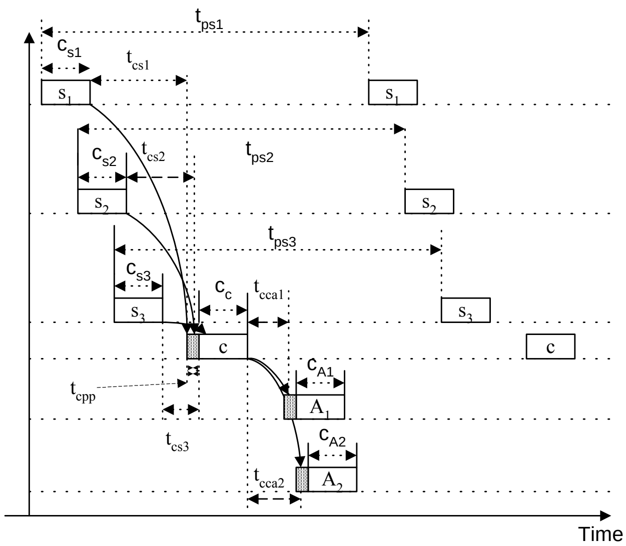

Figure 5. Time Diagram representation based upon a Computer Network.

Inconsistencies appear during communication and consumption times, which as well as the jitter, play an important role.

For this case study, Table 1 shows the consumptions and periods of the involved nodes, where sensors and actuators are organized according to an EDF algorithm. Also, Table 2 shows some related time delays, according to Figure 5 and its variations.

TABLE 1. CONSUMPTION TIMES AND PERIODS FROM TASKS (IN SECONDS)

Component	Consumption time	Variation	Time Deadlines	Variation
S_1	C_{s1} = 0.03	6-8%	0.08	2-3%
S_2	C_{s2} =0.03	6-8%	0.08	2-3%
S_3	C_{s3} =0.03	6-8%	0.08	2-3%
C	C_C = 0.05	7-9%	0.08	2-3%
A_1	C_{A1} = 0.045	5-7%	0.08	2-3%
A_2	C_{A2} = 0.045	5-7%	0.08	2-3%

TABLE 2. COMMUNICATION TIMES NEEDED (IN SECONDS)

Communication time	Spent time
t_{cs1}	0.02
t_{cs2}	0.02
t_{cs3}	0.02
t_{cca1}	0.02
t_{cca2}	0.02
jitter	0.015

Based upon these conditions, scheduling modifications are performed. Such scheduling modifications affect the performance response of peripheral elements, where B and C matrices are altered, where time delays are defined as follows (Table 3):

$$\tau_1^1 = 0, \ \tau_2^1 = t_{cca1} + jitter + variation \text{ and } \tau_3^1 = 0.08$$

$$\tau_1^2 = 0, \ \tau_2^2 = t_{cca2} + jitter + variation \text{ and } \tau_3^2 = 0.08$$

TABLE 3. FUZZY LOGIC RULES CONSIDERING THE RELATED PLANT

Rule	Linealized Plant
If x_1 is A_{11} and x_2 is A_{12} and x_3 and A_{13}	$x(k+1) = a_1^p x(k) + B_1^p u$
If x_1 is A_{21} and x_2 is A_{22} and x_3 and A_{23}	$x(k+1) = a_2^p x(k) + B_2^p u$
If x_1 is A_{31} and x2 is A_{32} and x_3 and A_{33}	$x(k+1) = a_3^p x(k) + B_3^p u$

Therefore, control reconfiguration becomes necessary in order to keep certain response level. For this case study, three different control laws are proposed, as shown in Table 4.

TABLE 4. FUZZY LOGIC RULES CONSIUDERING THE RELATED CONTROL LAW

Rule	Control Law
If x_1 is A_{11} and x_2 is A_{12} and x_3 and A_{13}	$U=g_1X$
If x_1 is A_{21} and x_2 is A_{22} and x_3 and A_{23}	$U=g_2X$
If x_1 is A_{31} and x2 is A_{32} and x_3 and A_{33}	$U=g_3X$

The type of membership functions are Gaussian functions, normally distributed over the rank of each state, as shown in Figure 6. In case that any state is gone further, its effect is reduced through the related control law.

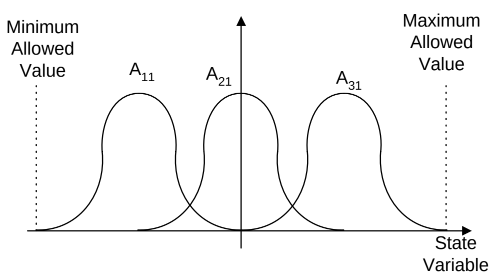

Figure 6. State Variable.

5. Second Case Study

The second case study presented in this chapter is a 2-DOF helicopter system, integrated to a CanBus network. A more detailed information about this system can be found in Esquivel-Flores et al. (2010). The sampling period of sensor tasks for the NCS is set to 1 ms. Controller and actuator tasks are event driven.

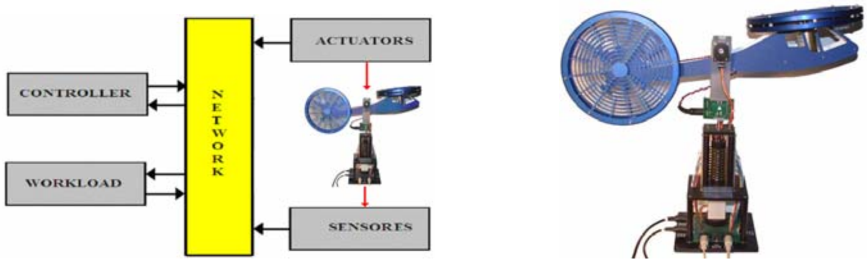

Figure 7. Networked Control System with workload.

This case study results to be a MIMO nonlinear, open-loop unstable, and time varying system. The actual 2-DOF Helicopter implementation consists of a helicopter model mounted on a fixed base with two propellers, powered by DC motors, as shown in Figure 7. The front propeller controls the elevation of the helicopter nose about the pitch axis, and the back propeller controls the side to side motions of the helicopter about the yaw axis. The pitch and yaw angles are measured using high-resolution encoders. The pitch encoder and motor signals are transmitted via a slip ring. This eliminates the possibility of wires tangling on the yaw axis, and allows the yaw angle to rotate freely about 360 degrees.

5.1 Numerical simulations

Numerical simulations of the helicopter system have been developed, without control and with LQR controller, for values of maximum, minimum, and real frequencies as described in Table 5.

TABLE 5. Maximum, minimum, and real frequencies.

Node	Max freq.	Min. freq.	Real freq
1	60	40	55
2	50	30	50
3	50	10	25
4	45	25	30

For this case study, matrix A and its related eigenvalues are:

$$A = \begin{bmatrix} 0.1250 & 0.7500 & 0.2500 & 0.6250 \\ 1.3333 & 0.1667 & 0.3333 & 0.8333 \\ 4.0000 & 3.0000 & 0.5000 & 2.5000 \\ 1.6000 & 1.2000 & 0.4000 & 0.2000 \end{bmatrix} \qquad \begin{array}{l} \lambda_1 = 3.2536 \\ \lambda_2 = -0.5885 \\ \lambda_3 = -0.8585 \\ \lambda_4 = -0.8150 \end{array}$$

5.2 LQR Control

For this case study, $Q, R \in \Re^{4x4}$ are weight matrices, chosen as follows:

$$Q = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The gain $K \in \Re^{4x4}$ and $A_c = (A - BK) \in \Re^{4x4}$ are set to:

$$K = \begin{bmatrix} 137.32 & 103.95 & 4042 & 87.32 \\ 1124.75 & 94.62 & 34.41 & 79.38 \\ 45.42 & 34.41 & 12.70 & 28.90 \\ 130.98 & 99.23 & 36.12 & 83.5 \end{bmatrix} \qquad A_c = \begin{bmatrix} -2.3251 & -1.10360 & -0.4237 & -0.9347 \\ -1.3398 & -1.8586 & -0.4017 & -0.8683 \\ 3.0298 & 2.2650 & 0.2332 & 1.8824 \\ -1.1727 & -0.9008 & -0.3624 & -1.5650 \end{bmatrix}$$

The eigenvalues of A_c are $\lambda_1 = -3.2536$, $\lambda_2 = -0.5997$, $\lambda_3 = -0.8606$, $\lambda_4 = -0.8182$
This makes that the frequencies can be managed into region L.

5.3 Implementation Approach

The implementation of this frequency transition model is included into the helicopter model, simulated on True-Time (Cervin et al., 2003; Esquivel-Flores et al. 2010). Four sensors sample pitch, yaw, pitch derivative, and yaw derivative signals, with period of 0.030 ms. Figure 8 shows the system instability for the angles pitch, yaw, and derivatives through 25 s.

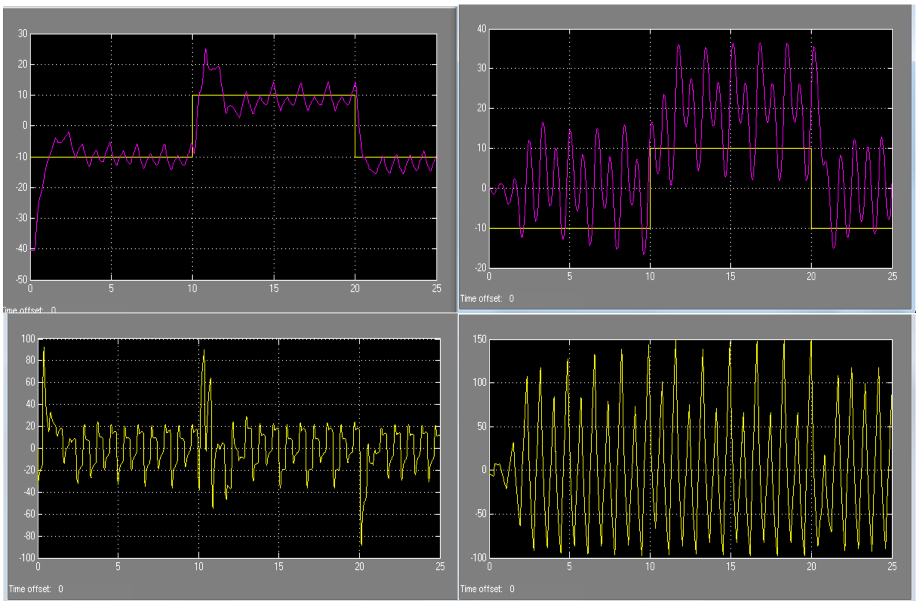

Figure 8. Response of 2-DOF Helicopter. Pitch and Yaw angles and derivatives.

Using the system frequency transition model (starting in the second 10), the system becomes stable due to change in the frequency data transmission of the sensors, as shown in Figure 9.

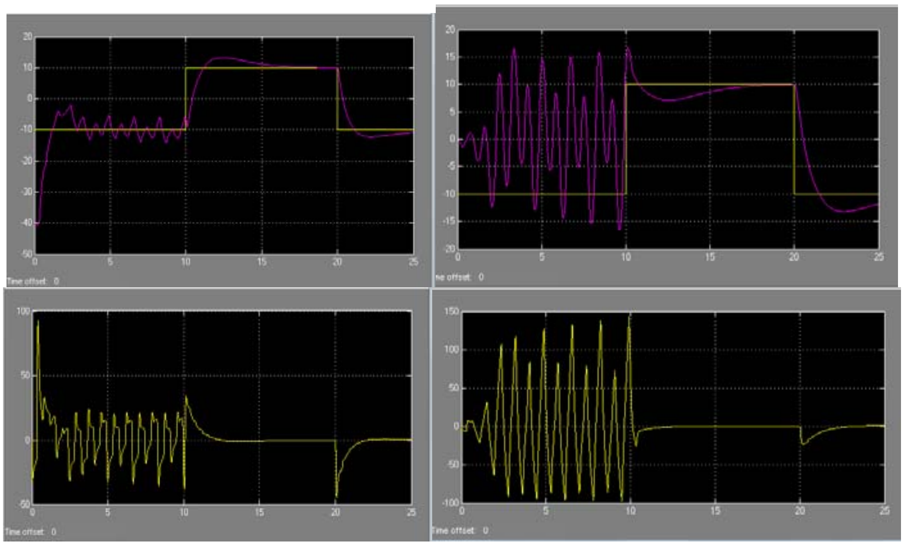

Figure 9. Response of 2-DOF Helicopter. Pitch and Yaw angles and derivatives using frequency transition model.

Notice that the frequency controller modifies the frequencies into the limits defined by minimal and maximal frequencies.

6. Conclusions

NCSs present a high-order, nonlinear behavior, that is understandable in terms of the unpredictable response of a distributed system. In such a system, time delays may be bounded, as result of certain systematic behavior. Here, frequency transmission bounding is tested and proven a suitable approach, since information is delimited in terms of a stable situation. Since NCSs are controlled discretely, a fuzzy control approach is presented, having a valid response due to it has been coupled with multi-time delays, and thus, getting a global stable behavior in a local manner.

The results presented in this chapter allow fully understanding of the nonlinear behavior from computer network, as well as the universal approximation inherent to fuzzy control in terms of a bounded strategy.

Acknowledgments: Authors would like to thank the support of project PAPIIT UNAM (IN103310).

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